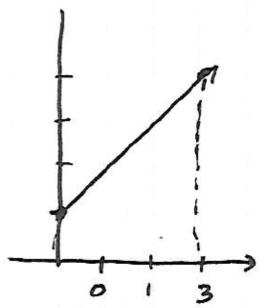
AP Calculus AB

Exact Area Under a Curve

1) $f(x) = x+1$; $a=0$, $b=3$



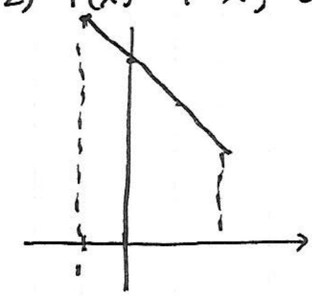
$\int_0^3 (x+1)\,dx$

$\left[\frac{1}{2}x^2 + x + C\right]\Big|_0^3$

$\left[\frac{1}{2}(9)+3\right] - [0]$

$\boxed{\frac{15}{2}}$

2) $f(x) = 4-x$; $a=-1$, $b=2$



$\int_{-1}^2 (4-x)\,dx$

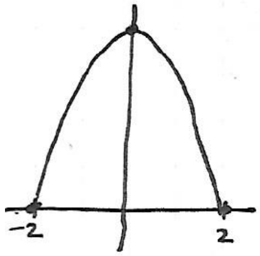
$\left[4x - \frac{1}{2}x^2 + C\right]\Big|_{-1}^2$

$[8-2] - \left[-4-\frac{1}{2}\right]$

$(6) - \left(-\frac{9}{2}\right)$

$\boxed{\frac{21}{2}}$

3) $f(x) = 4-x^2$



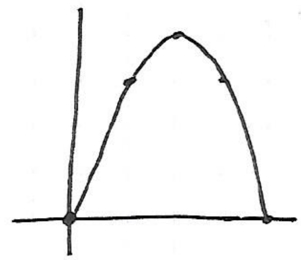
$\int_{-2}^2 (4-x^2)\,dx$

$\left[4x - \frac{1}{3}x^3 + C\right]\Big|_{-2}^2$

$\left[8 - \frac{1}{3}(8)\right] - \left[-8 + \frac{1}{3}(8)\right]$

$16 - \frac{16}{3} = \boxed{\frac{32}{3}}$

4) $f(x) = 4x - x^2$



$\int_0^4 (4x - x^2)\,dx$

$\left[2x^2 - \frac{1}{3}x^3 + C\right]\Big|_0^4$

$\left[32 - \frac{64}{3}\right] - [0]$

$\boxed{\frac{32}{3}}$

5) $f(x) = \cos x$; $a = -\frac{\pi}{2}$, $b = \frac{\pi}{2}$

$\int_{-\pi/2}^{\pi/2} \cos x\,dx = 2$

6) $f(x) = \sin x$; $a = \frac{\pi}{6}$, $b = \frac{\pi}{3}$

$\int_{\pi/6}^{\pi/3} \sin x\,dx = 0.366$

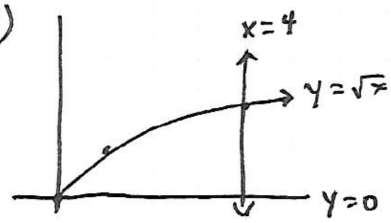
7) $f(x) = e^{2x}$; $a=0$, $b=1$

$\int_0^1 e^{2x}\,dx = 3.195$

8) $f(x) = e^x$; $a=-1$, $b=1$

$\int_{-1}^1 e^x = 2.350$

9)



$\int_0^4 \sqrt{x}\,dx = \int_0^4 x^{1/2}\,dx$

$\left[\frac{2}{3}x^{3/2} + C\right]\Big|_0^4$

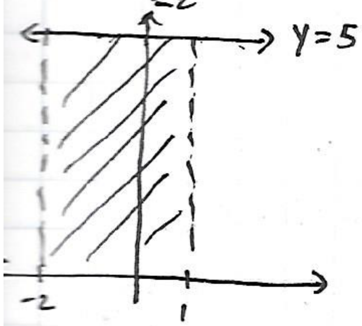
$\frac{2}{3}(8) = \boxed{\frac{16}{3}}$

10) $\lim_{n\to\infty} \sum_{k=1}^{n} x_k^2 \Delta x_k \rightarrow \int_0^2 x^2\,dx$

11) $\lim_{n\to\infty} \sum_{k=1}^{n} (x_k^2 - 3x_k)\Delta x_k \rightarrow \int_{-7}^5 (x^2-3x)\,dx$

12) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{x_k}\Delta x_k \rightarrow \int_1^4 \frac{1}{x}\,dx$

13) $\int_{-2}^{1} 5\,dx = 3(5) = 15$



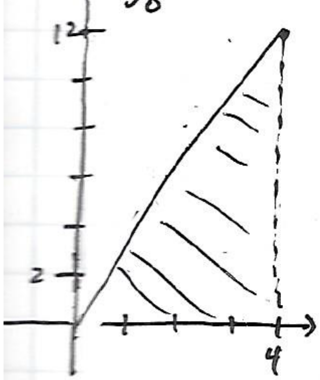
14) $\int_{3}^{7} (-20)\,dx$

$= (-20) \cdot 4$

$= -80$

15) $\int_{0}^{4} 3\theta\,d\theta = \frac{1}{2}(4)(12)$

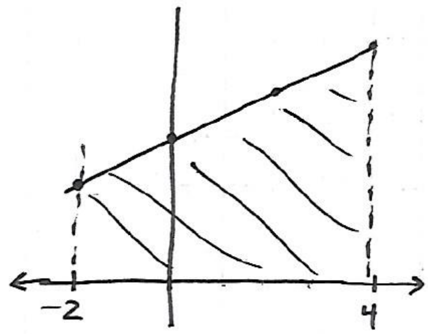
$= \boxed{24}$



16) $\int_{-2}^{4} \left(\frac{1}{2}x+3\right) = \frac{1}{2}\left[2+5\right] \cdot 6$

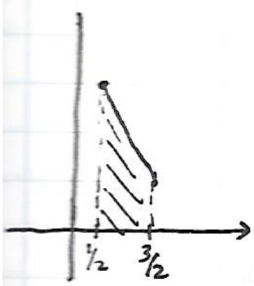
$= 3(7)$

$= \boxed{21}$



17) $\int_{1/2}^{3/2} (-2x+4)\,dx = \frac{1}{2}\left[3+1\right] \cdot 1$

$= \boxed{2}$



18) $\int_{-3}^{3} \sqrt{9-x^2}\,dx = \frac{1}{2}\pi(3)^2$

$= \boxed{\frac{9\pi}{2}}$